

Lorentz Violation of The Standard Model

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(Dated: February 2, 2008)

Though the standard model of electroweak interactions[1, 2, 3] is commonly believed to provide a successful unification of electromagnetic and weak interactions, the approximation in the massless limit and the assumption of massless fermion in this model should be investigated further with rigorousness. Here it will be shown that this approximation violates Lorentz invariance and it is still necessary even with the assumption of a massless fermion and the Higgs mechanism[4, 5]. We conclude that the unification of electroweak interactions is only valid with the assumption of Lorentz violation.

PACS numbers: 12.15.-y, 11.30.Cp

I. INTRODUCTION

The standard model of the Glashow-Weinberg-Salam theory, suggested as a unified model of electroweak interactions, is based on the massless approximation and the assumption of massless fermion, not just that of massless neutrino to be accurate. Here the physical argument of electroweak unification will be investigated after clarifying the ambiguity in the helicity and chirality of fermion following the standard textbook[6]

II. HELICITY AND CHIRALITY

A Dirac field ψ can be written as a linear combination of plane waves since it obeys the Klein-Gordon equation.

$$\psi(x) = u(p)e^{-ip \cdot x}$$

where $p^2 = m^2$ and let us consider solutions with positive frequency, $p^0 > 0$, for simplicity. One may obtain the solutions of the Dirac equation in the rest frame, $p = p_0 = (m, 0)$

$$u(p_0) = \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

for any numerical two-component spinor ξ . The general form of $u(p)$ in other frame can be derived by boosting $u(p_0)$ in the rest frame. Let us define η , *rapidity* by considering a boost along the z -direction to the 4-momentum vector. For finite η ,

$$\begin{aligned} \begin{pmatrix} E \\ p^3 \end{pmatrix} &= \exp \left[\eta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right] \begin{pmatrix} m \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} m \cosh \eta \\ m \sinh \eta \end{pmatrix} \end{aligned}$$

Applying the same boost to $u(p_0)$,

$$u(p) = \exp \left[-\frac{1}{2} \eta \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \right] \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

$$= \begin{pmatrix} \left[\sqrt{E+p^3} \left(\frac{1-\sigma^3}{2} \right) + \sqrt{E-p^3} \left(\frac{1+\sigma^3}{2} \right) \right] \xi \\ \left[\sqrt{E+p^3} \left(\frac{1+\sigma^3}{2} \right) + \sqrt{E-p^3} \left(\frac{1-\sigma^3}{2} \right) \right] \xi \end{pmatrix}$$

If $\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (spin up along the z -axis) in the boosted frame of z direction,

$$u_{z+;h+} = \begin{pmatrix} \sqrt{E-p^3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \sqrt{E+p^3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix}$$

while for $\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (spin down along the z -axis) in the boosted frame of z direction we have

$$u_{z+;h-} = \begin{pmatrix} \sqrt{E+p^3} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sqrt{E-p^3} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix}$$

Following the same steps, in the boosted frame of $-z$ direction we have

$$\begin{aligned} u_{z-;h+} &= \begin{pmatrix} \sqrt{E-p^3} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sqrt{E+p^3} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \\ u_{z-;h-} &= \begin{pmatrix} \sqrt{E+p^3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \sqrt{E-p^3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \end{aligned}$$

where *helicity* h_{\pm} is defined by the momentum of fermion and its spin orientation: if spin orientation is in the same direction as its momentum, it is called right-handed helicity ($h+$). Therefore, a massive fermion field of the Dirac equation can be described by two nonzero Weyl spinors with left-handed and right-handed chirality

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

where $\psi_{L,R} \neq 0$ for a massive fermion and *chirality* (L, R) is defined to indicate either of these two-component objects. However, the chirality is not a physical observable

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unlike the helicity since no corresponding physical measurement is available and also a massive fermion satisfying the Dirac equation cannot be represented by only one chirality since it only denotes a part of the solution to the Dirac equations are there are always two chiralities for a massive particle ($\psi_{L,R} \neq 0$). The helicity of a massive particle is also not well defined in the rest frame where its momentum is zero and the left-handed and right-handed particle are indistinguishable. It is also not a fundamental property of particle since it is not conserved under Lorentz transformations: one can always find a boosted frame, where the helicity is opposite due to its reversed momentum, for example, $u_{z+;h+}$ can be observed as $u_{z-;h-}$ in a boosted reference frame. Therefore, the helicity of a massive fermion is not a Lorentz-invariant observable and it cannot be considered as a fundamental property of particle.

A massless fermion can be derived by taking the massless limit or solving massless Dirac equation. For the boosted frame of z direction we have

$$u_{z+;h+} = \sqrt{E+p^3} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u_{z+;h-} = \sqrt{E+p^3} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

In the boosted frame of $-z$ direction,

$$u_{z-;h+} = \sqrt{E+p^3} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad u_{z-;h-} = \sqrt{E+p^3} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Unlike massive fermions, they can be represented by only one chirality

$$\Psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

Since a massless fermion has neither the rest frame nor a boosted frame which reverses its helicity, the helicity is well defined and can be considered as a fundamental property of particle that is Lorentz invariant. Its chirality also can be considered as fundamental property as it corresponds to its helicity, which provides the physical measurement of chirality.

The distinction of massive and massless particles in chirality and helicity should be treated with care in approximating a massive fermion in the massless limit. A massive fermion in a large boost can be approximately described as a massless particle. However, the exact solutions of the massive Dirac equation in the rest frame cannot be achieved by Lorentz transformations of the approximated ones back to the rest frame, for example,

$$\begin{aligned} u &= \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix} && \text{in the rest frame} \\ &\rightarrow \begin{pmatrix} \sqrt{E+p^3}\xi \\ \sqrt{E-p^3}\xi \end{pmatrix} && \text{in the boosted frame} \end{aligned}$$

$$\begin{aligned} &\rightarrow \sqrt{2E} \begin{pmatrix} \xi \\ 0 \end{pmatrix} && \text{approximation} \\ &\rightarrow \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix} && \text{back to the rest frame} \end{aligned}$$

Therefore, the massless approximation violates Lorentz invariance and thus it should not be justified to determine the fundamental property of a massive particle as that of massless one.

III. THE STANDARD MODEL

The weak interactions have a certain structure of $(1 \pm \gamma^5)[7, 8]$ as

$$H_{weak} = \bar{\psi} \gamma^\mu \frac{(1 \pm \gamma^5)}{2} \psi,$$

while the electromagnetic interactions are

$$H_{em} = \bar{\psi} \gamma^\mu \psi$$

In the standard model, they are unified as the electroweak interactions in

$$H_{ew} = \bar{\psi}_{L,R} \gamma^\mu \psi_{L,R}$$

as massive fermions are represented with definite helicities in the massless limit[1, 2, 3]. This model suggests to interpret $\frac{1}{2}(1 \pm \gamma^5)$ not as a structure of interactions, but as a physical operator acting on fermion fields so that the weak interactions are

$$\begin{aligned} H_{weak} &= \bar{\psi} \gamma^\mu \frac{(1 \pm \gamma^5)}{2} \psi \\ &= \bar{\psi} \gamma^\mu \left[\frac{(1 \pm \gamma^5)}{2} \right]^2 \psi \\ &= \bar{\psi} \frac{(1 \mp \gamma^5)}{2} \gamma^\mu \frac{(1 \pm \gamma^5)}{2} \psi \\ &= \bar{\psi}_{L,R} \gamma^\mu \psi_{L,R} \end{aligned}$$

where

$$\begin{aligned} \psi_L &= \frac{1}{2}(1 - \gamma^5)\psi = \begin{pmatrix} \psi'_L \\ 0 \end{pmatrix} \\ \psi_R &= \frac{1}{2}(1 + \gamma^5)\psi = \begin{pmatrix} 0 \\ \psi'_R \end{pmatrix} \end{aligned}$$

However, no massive fermion of the Dirac equation corresponds to $\psi_{L,R}$ and thus it is only valid for a massive particle in the massless limit. We may argue that the calculation of H_{weak} would be the same regardless of this approximation because $\frac{1}{2}(1 \pm \gamma^5)$ associates with only one chiral component.

$$\begin{aligned} H_{weak} &= \bar{\psi} \gamma^\mu \frac{(1 \pm \gamma^5)}{2} \psi \\ &= \bar{\psi}_{h\pm} \gamma^\mu \frac{(1 \pm \gamma^5)}{2} \psi_{h\pm} \\ &\rightarrow \bar{\psi}_{L,R} \gamma^\mu \frac{(1 \pm \gamma^5)}{2} \psi_{L,R} \\ &= \bar{\psi}_{L,R} \gamma^\mu \psi_{L,R} \end{aligned}$$

However, we have two distinctive structure $\frac{1}{2}(1 \pm \gamma^5)$ for a fermion field since whether the particle is left- or right-handed in some reference frames should not determine which of these two structures we have in the rest frame and thus the massless approximation is also required for this argument.

$$\frac{1}{2}(1 - \gamma^5)\psi_{h+} \neq \frac{1}{2}(1 - \gamma^5)\psi_R = 0$$

while

$$\frac{1}{2}(1 + \gamma^5)\psi_{h+} = \frac{1}{2}(1 + \gamma^5)\psi_R$$

The electromagnetic interactions are also approximated since a massive Dirac fermion that we observe is not $\psi_{L,R}$, but $\psi_{h\pm}$.

$$\begin{aligned} H_{em} &= \bar{\psi}\gamma^\mu\psi \\ &= \bar{\psi}_{h\pm}\gamma^\mu\psi_{h\pm} \\ &\rightarrow \bar{\psi}_{L,R}\gamma^\mu\psi_{L,R} \end{aligned}$$

Therefore, the unification of electroweak interactions is based on the approximation that violates Lorentz invariance.

The most accurate description of electroweak interactions should be obtained in the rest frame where the exact solutions of the Dirac equations are available and always the same whether they are observed as left- or right-handed in other reference frames since the interactions should be Lorentz invariant. In the rest frame, there are still two different electroweak interactions.

$$\begin{aligned} H_{ew} &= \bar{\psi}_0\gamma^\mu\frac{(1 \pm \gamma^5)}{2}\psi_0 \\ &\neq \bar{\psi}_0\gamma^\mu\psi_0 \end{aligned}$$

Therefore, the unification of electroweak interactions fails in the most accurate description of interactions and it is only valid with the assumption of Lorentz violation.

IV. THE HIGGS MECHANISM

This Lorentz violation of the standard model might be avoided by the assumption that a fermion is massless

as its mass is obtained from the spontaneous symmetry breaking[4, 5]. In the Higgs mechanism, a fermion is defined as a massless particle with a definite chirality $\psi_L = \frac{1}{2}(1 - \gamma^5)\psi$. However, it would be inconsistent if this definition of massless fermion remains valid even after its mass is acquired since a massive fermion satisfying the Dirac equation cannot be represented by only one chirality. When we describe the physics of a massive fermion, its nonzero mass should be accepted as a given physical property of particle in the fundamental equations that is independent of any proceeding physical events that occurred before: a fermion field should be defined by the massive Dirac equation regardless of how its mass is given. Since the Higgs mechanism that explains how mass is acquired is only a hypothetical event with no physical observations for its verification available, the given physical property of a massive particle should be the same after the Higgs mechanism and thus the fermion should be redefined with mass in order to be consistent. Otherwise, the fundamentality of the Dirac equation and mass would be contradicted since the helicity is obtained from the massless Dirac equation while other fundamental properties such as the definition of antiparticle from the massive Dirac equation. Therefore, the assumption of a massless fermion is invalid to describe a massive fermion after the spontaneous symmetry breaking and thus the Lorentz-violating approximation is still required for the unification of electroweak interactions.

V. CONCLUSION

The massless approximation of the standard model proved to violate Lorentz invariance as the exact fermion field in the rest frame cannot be obtained from Lorentz transformations on the approximated one. The assumption of massless fermion with the Higgs mechanism fails to eliminate the necessity of this approximation since the fermion should be redefined as a massive particle after its mass is acquired. In conclusion, the unification of electroweak interaction in the standard model is only valid with the assumption of Lorentz violation.

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- [1] S. Weinberg, Phys. Rev. Lett. **19** (1967), 1264.
 - [2] A. Salam, in *Elementary Particle Theory (Nobel Symposium No. 8*, ed. N. Svartholm, Stockholm, 1968) 367.
 - [3] S. Glashow, Nucl. Phys. **22**, 579 (1961).
 - [4] P.W. Higgs, Phys. Rev. **145** (1966) 1156.
 - [5] T.W.B. Kibble, Phys. Rev. **155** (1967) 1554.
 - [6] Michael E. Peskin and Daniel V. Schoreder, *An introduc-*

tion to Quantum Field Theory, Addison-Wesley Publishing Co., (1995)

- [7] R.P. Feynman and M. Gell-Mann, Phys. Rev. **109** (1958) 193.
- [8] E.C.G. Sudarshan and R.E. Marshak, Phys. Rev. **109** (1958) 1860.